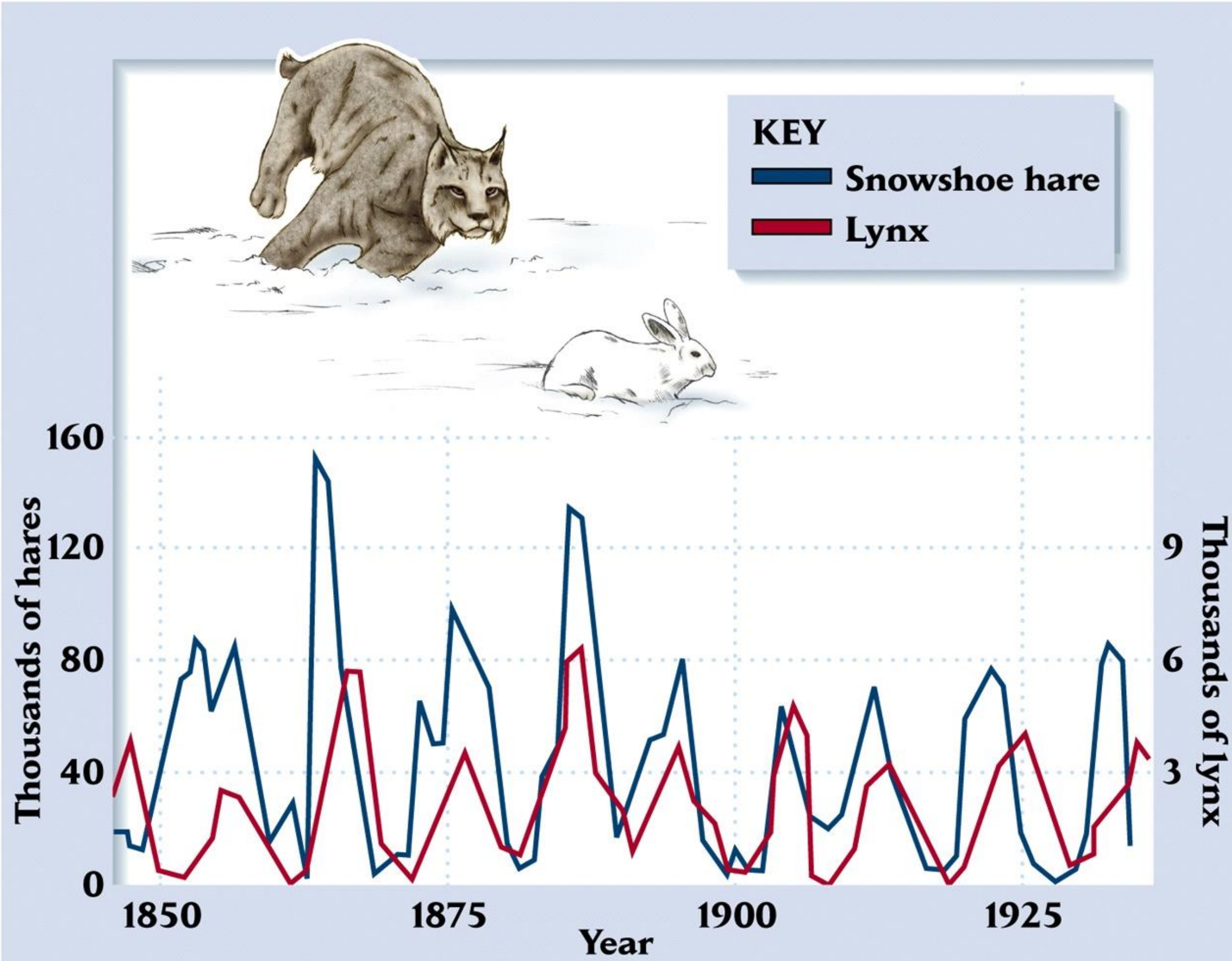
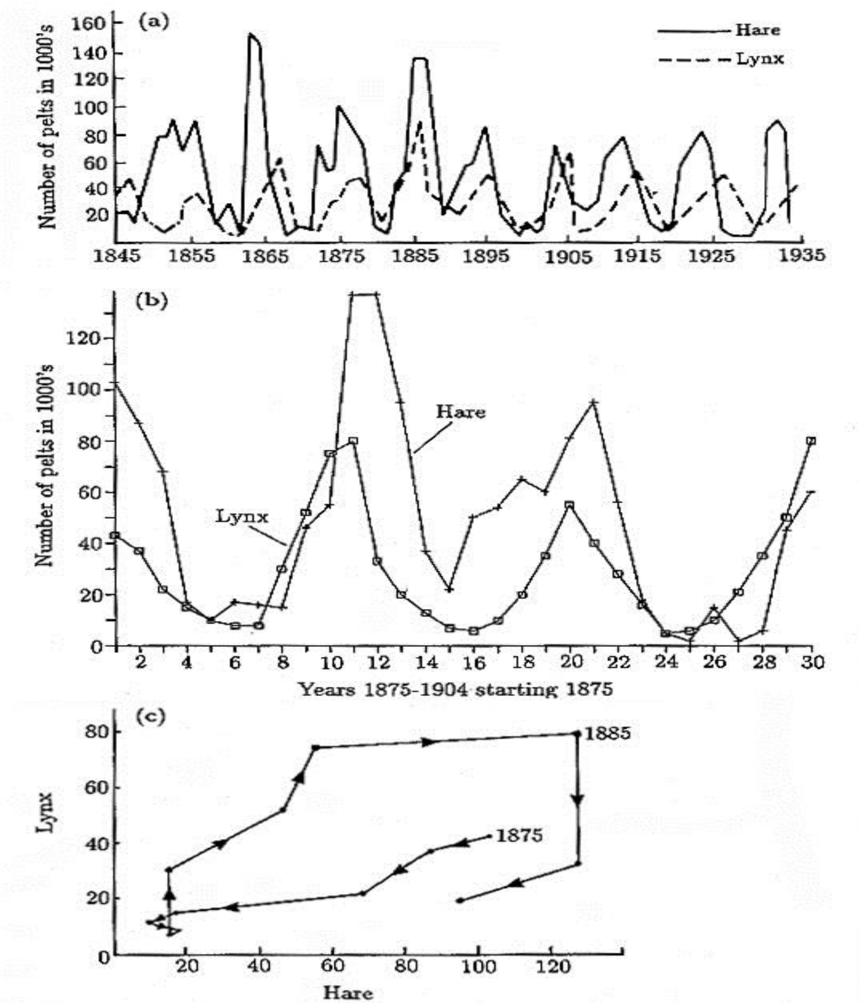
Modelowanie dynamiki populacji – drapieżniki i ofiary


KEY
Snowshoe hare
Lynx
160
120
80
40
0
Thousands of hares
9
6
3
Thousands of lynx
1850
1875
1900
1925
Year

Hare-Lynx data (Canada)



- **A verbal model of predator-prey cycles:**

1. Predators eat prey and reduce their numbers
2. Predators go hungry and decline in number
3. With fewer predators, prey survive better and increase
4. Increasing prey populations allow predators to increase

- And repeat…

Generic Model

$$\frac{dx}{dt} = f(x) - h(x,y)$$

$$\frac{dy}{dt} = -g(y) + eh(x,y)$$

- f(x) prey growth term
- g(y) predator mortality term
- h(x,y) predation term
- e prey into predator biomass conversion coefficient

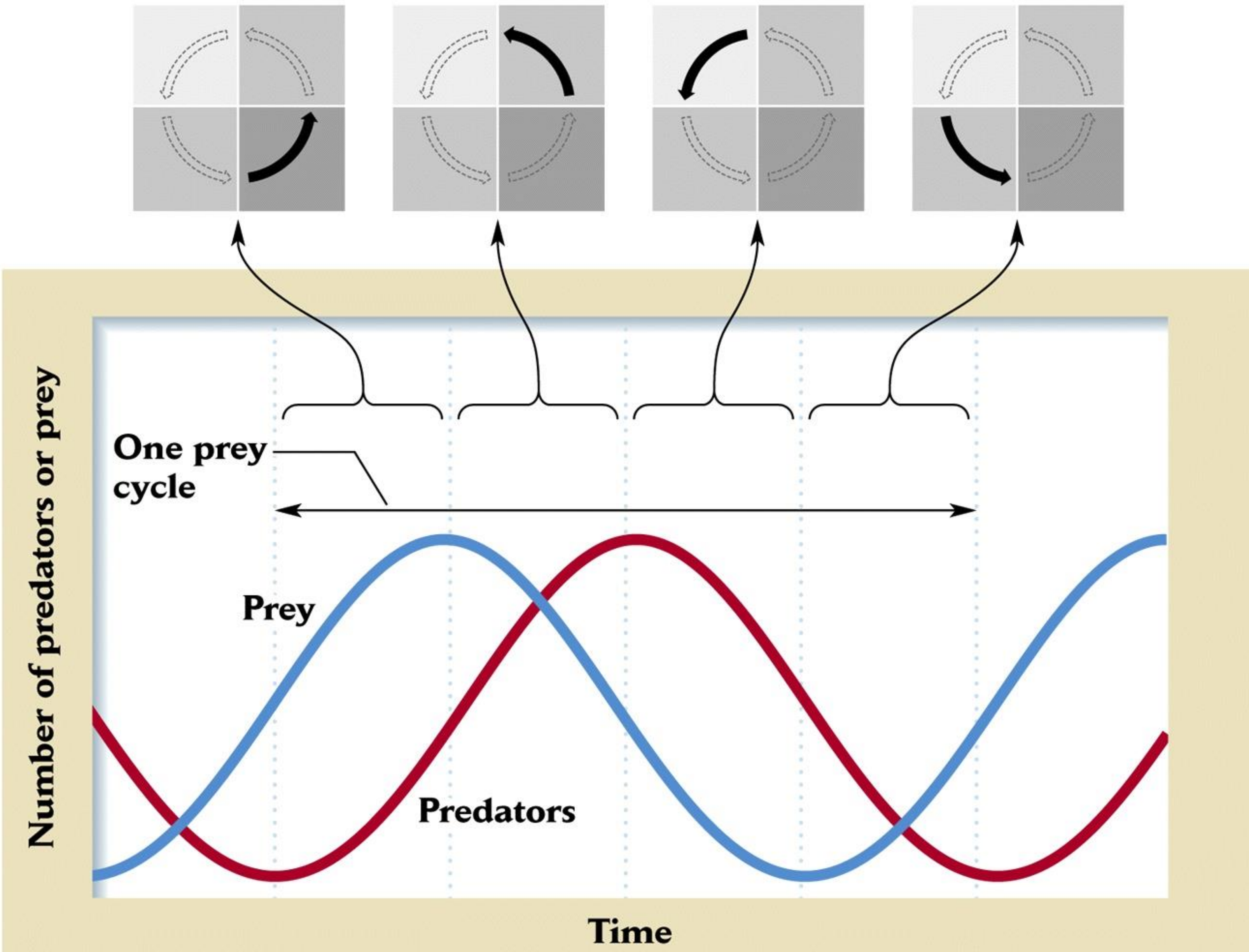
Lotka-Volterra Model

$$\frac{dx}{dt} = rx - axy$$

$$\frac{dy}{dt} = -my + bxy$$

- r prey growth rate : Malthus law
- m predator mortality rate : natural mortality
- Mass action law
- a and b predation coefficients : b=ea
- e prey into predator biomass conversion coefficient

Lotka-Volterra: solution



Other effects:

Time delay:

$$\dot{N}(t) = \frac{\mathrm{d}N}{\mathrm{d}t} = rN(t)\left(1 - \frac{N(t-T)}{K}\right)$$

(Hutchinson model)

Behaviour: when rK is sufficiently large N(t) oscillates (single species model!)

Problem:

Consider the predator–prey model defined by the following system of two recurrence equations:

$$H_{n+1} = aH_n(1 - H_n) - bH_nP_n$$
$$P_{n+1} = dH_nP_n,$$

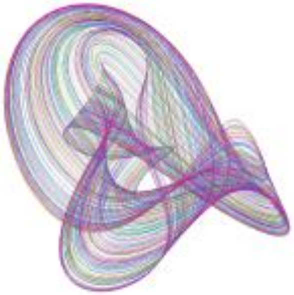
where H_n and P_n are the respective reduced prey and predator population densities, and a, b and d are positive constant. This model assumes that the predator can survive only in the presence of prey. Find the fixed points, and discuss their stability.

3-species model

- X’= ax-bxy (prey-- mouse)
- Y’= -cy+dxy-eyz (predator-- snake)
- Z’= -fz+gxz (super-predator-- owl)
  - **a**: natural growth rate of prey in the absence of predation
  - **b, e,**: death rate due to predation
  - **c, f**: natural death rate of predator
  - **d, g**: growth rate due to predation

Poincaré-Bendixson

- No chaos in d=2 systems: trajectory approaches either a fixed point or a closed orbit
- In d>2 systems trajectories might settle on strange attractors



Limitations of Lotka-Volterra

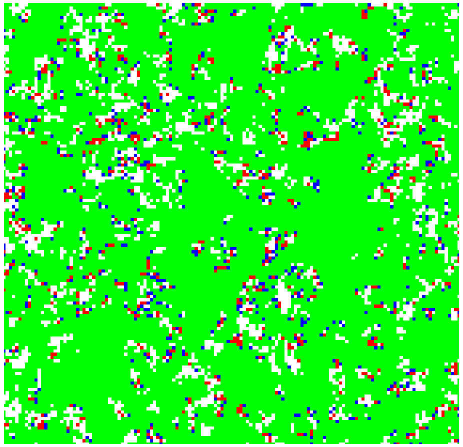
- No time lag between predator and prey population responses
- Lack of heterogeneities in the distribution of species

$$\frac{\partial n}{\partial t} = rn(1 - n / K) + D\frac{\partial^2 n}{\partial x^2}$$

- Neutrally Stable= there is no attraction to some equilibrium point
  Any perturbation in the model will have it continue to cycle at a new amplitude until a new force acts on the model
  Eventually one of the populations oscillations may reach an axis signaling that a population has died out

- Based off of a Type I Functional response which is the least realistic type of functional response
  Type I response does not have any density dependence factors, therefore predator populations are linearly related to prey populations without any consideration of the density of the predator population

Prey – predators: lattice version



Rules:

- Choose a site
- Update if prey (breeds if possible)
- Update if predator (breeds,consumes prey if around, dies of hunger)
- Do nothing if empty

individual-oriented dynamics, spatial and temporal heterogeneities

Mean-field approximation:

$$\frac{dx(t)}{dt} = rx(t)[1 - x(t)^{2d}] - (1-r)x(t)y(t),$$

$$\frac{dy(t)}{dt} = (1-r)x(t)y(t)[1 - y(t)^{2d}] - (1-r)[1 - x(t)]y(t).$$